Nonlinear Systems and Control

Lecture 1

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Department of Mechatronics Engineering – Çankaya University

Master Course in Electronic and Communication Engineering
Credits (3/0/3)

Content and Structure

Content

- Properties of nonlinear systems
- Stability analysis
- Passivity
- Feedback linearization
- Flatness-based control
- Backstepping
- Passivity-based control

Structure

- 3 lecture hours on Monday: 18:40 – 21:30
- Exercises sheets during the lecture
- Office hours: Tuesday, 14:30 - 15:30

Grading and Literature

Grading

- 10 Quizzes (15%)
- 1 Midterm Exam (35%)
- 1 Final Exam (50%)

Literature

- Hassan K. Khalil: "Nonlinear Systems", Prentice Hall, 2002 (ISBN: 0-13-067389-7) (Main Textbook)
- Alberto Isidori: "Nonlinear Control Systems", Springer, 1995 (ISBN: 3-54-019916-0)
- Horacio J. Marquez: "Nonlinear Control Systems: Analysis and Design", Wiley-Interscience, 2003 (ISBN: 0-471-42799-3)
- Eduardo D. Sontag: "Mathematical Control Theory: Deterministic Finite Dimensional Systems", Second Edition, Springer, New York, 1998 (online: http://www.math.rutgers.edu/~sontag/FTP_DIR/sontag_mathematical_control_theory_springer98.pdf)

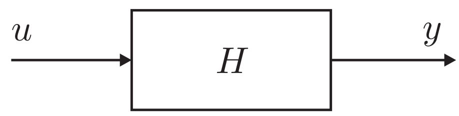
Linear Systems: Definition

System

- Input signal u
- Output signal y
- Operator H that maps u to y

$$y(t) = H\{u(t)\}$$

Illustration



Linear System

- Input signal as superposition of different input signals

$$u(t) = \alpha_1 u_1(t) + \alpha_2 u_2(t), \quad (\alpha_1, \alpha_2 \in \mathbb{R})$$

- Output signal as superposition of corresponding output signals

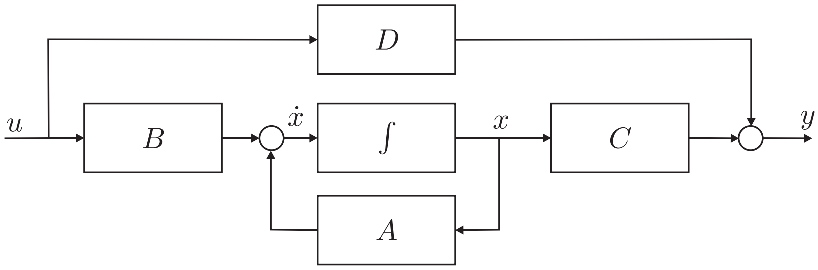
$$y(t) = H\{\alpha_1 u_1(t) + \alpha_2 u_2(t)\} = \alpha_1 H\{u_1(t)\} + \alpha_2 H\{u_2(t)\}$$

Linear Systems: State Space Models

State Equations

$$\dot{x} = A\,x + B\,u$$
$$y = C\,x + D\,u$$

Illustration



Signals

- Input $u(t) \in \mathbb{R}^m$
- Output $y(t) \in \mathbb{R}^p$
- State vector $x(t) \in \mathbb{R}^n$
- State derivative $\dot{x}(t) \in \mathbb{R}^n$

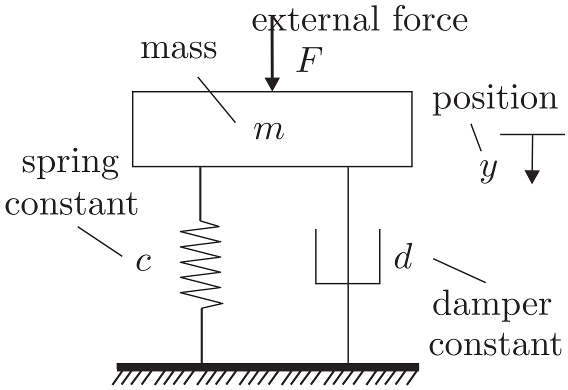
Matrices

- System matrix $A \in \mathbb{R}^{n\times n}$
- Input matrix $B \in \mathbb{R}^{n\times m}$
- Output matrix $C \in \mathbb{R}^{p\times n}$
- Feedthrough matrix $D \in \mathbb{R}^{p\times m}$

Linear Systems: State Space Model Example

Mass-Spring-Damper Example

- Spring force: $F_c = c\cdot y$
- Damper force: $F_d = d\cdot \dot{y}$
- Inertia force: $F_m = m\cdot \ddot{y}$
- Input: F
- Output: y



Gap 1

Linear Systems: State Space Model Example

Mass-Spring-Damper Example

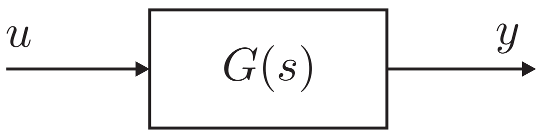
Gap 2

Linear Systems: Transfer Function Models

Transfer Function

$$Y(s) = G(s)\,U(s)$$

Illustration



Laplace Transform

- $y(t) \circ\!\!-\!\!\bullet\, Y(s)$
- $u(t) \circ\!\!-\!\!\bullet\, U(s)$

Computation from State Equations

$$G(s) = C(sI - A)^{-1}B + D$$

Properties of Transfer Functions

- Proper: relative degree is greater or equal to zero
- Stable: all poles lie in the open left half plane
- Oscillatory: there are conjugated complex poles

Linear Systems: Transfer Function Model Example

Mass-Spring-Damper Example

Gap 3

Linear Systems: Available Tools

Solvability

- Solution using the state space equation

$$y(t) = C(e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau) + D\,u(t)$$

- Solution using the transfer function

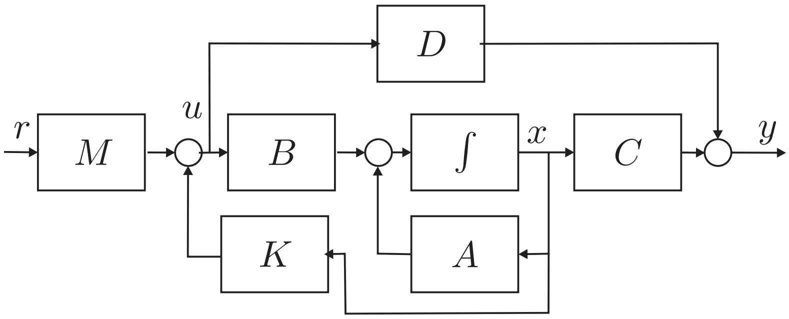
$$y(t) \circ\!\!-\!\!\bullet Y(s) = G(s)U(s)$$

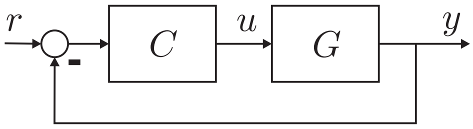
Important Conditions

- Controllability
- Observability

Linear Systems: Available Tools

Control Loop





Controller Design

- Linear state feedback control
- PID control, lead/lag compensator, etc.
- Bode plot design, root locus, Ziegler-Nichols method, etc.

Nonlinear Systems: Basics

General Model

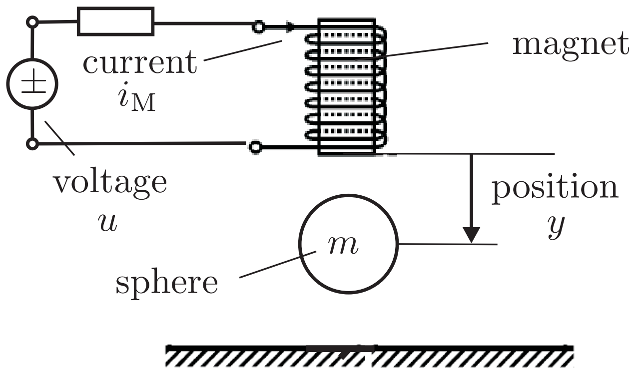
$$\dot{x} = f(t, x, u)$$
$$y = h(t, x, u)$$

- State vector: $x \in \mathbb{R}^n$
- Input: $u \in \mathbb{R}^m$
- Output: $y \in \mathbb{R}^p$
- Right-hand side (rhs): $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$
- Output function: $h : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$
  $\to$ f and h are usually assumed to be continuous functions
- n: state space order; m: number of inputs; p number of outputs

Nonlinear Systems: Example

Magnetic Suspension

Gap 4



Gap 5

Nonlinear Systems: Example

State Vector

- $x_1 = y$
- $x_2 = \dot{y}$
- $x_3 = i_{\mathrm{M}}$

State equations

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = g - \frac{k}{m}x_2 - \frac{\lambda\mu x_3^2}{2m(1+\mu x_1)^2}$$

$$\dot{x}_3 = \frac{1+\mu x_1}{\lambda}\Big(-Rx_3 + \frac{\lambda\mu x_3 x_2}{(1+\mu x_1)^2} + u\Big)$$

$$y = x_1$$

Gap 6

Nonlinear Systems: Example

Mobile Robot

Gap 7

Nonlinear Systems: Issues

Aim of this Course

- Solvability of the state equation
- Analysis
  - Stability
  - Controllability
  - Dynamics
- Controller design
  - Feedforward control
  - Feedback control
  - Modern design methods
- Examples

Next Lecture

- Existence of solutions for nonlinear state equations