Nonlinear Systems and Control

Lecture 10

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Feedback Connections: Notation

Block Diagram

Gap 1

State Equations

$$\dot{x}_1 = f_1(x_1, e_1) \qquad \dot{x}_2 = f_2(x_2, e_2)$$
$$y_1 = h_1(x_1, e_1) \qquad y_2 = h_2(x_2, e_2)$$
$$e_1 = u_1 - y_2 \qquad e_2 = u_2 + y_1$$

Feedback Connections: Overall System

Derivation

Gap 2

State Equations

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = f(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} u_1 \\ u_2 \end{bmatrix})$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = h(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} u_1 \\ u_2 \end{bmatrix})$$

Well-defined Connection if

$$e_1 = u_1 - h_2(x_2, e_2)$$
$$e_2 = u_2 + h_1(x_1, e_1)$$

has unique solution for $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

Feedback Connections: Example

Magnetic Valve

Gap 3

Feedback Connections: Asymptotic Stability

Theorem (Passivity in Feedback Connections)

Consider the feedback connection of two nonlinear systems as described before and assume that $f(x,u)$ is locally Lipschitz, $h(x,u)$ is continuous, $f(0,0)=0$ and $h(0,0)=0$. If both nonlinear systems are passive, then the feedback connection of both systems is also passive.

Proof

Gap 4

Feedback Connections: Passivity

Theorem (Asymptotic Stability in Feedback Connections)

Consider the feedback connection of two nonlinear systems as described before and assume that $f(x,u)$ is locally Lipschitz, $h(x,u)$ is continuous, $f(0,0)=0$ and $h(0,0)=0$. If both nonlinear systems are either strictly passive or output strictly passive and zero-state detectable, then the origin of the feedback system $\dot{x}=f(x,0)$ is asymptotically stable.

Proof

Gap 5

Feedback Connections: $\mathcal{L}_2$-Stability

Theorem ($\mathcal{L}_2$-Stability in Feedback Connections)

Consider the feedback connection of two nonlinear systems. Assume that $f(x,u)$ is locally Lipschitz, $h(x,u)$ is continuous, $f(0,0) = 0$, $h(0,0) = 0$. Assume that both nonlinear systems are output strictly passive ($e_1^T y_1 \geq \dot{V}_1 + \delta_1 y_1^T y_1$ for the first system and $e_2^T y_2 \geq \dot{V}_2 + \delta_2 y_2^T y_2$ for the second system). Then, the feedback connection is $\mathcal{L}_2$-stable and its $\mathcal{L}_2$-gain is less than or equal to $1/\min\{\delta_1, \delta_2\}$.

Proof

Gap 6

Feedback Connections: $\mathcal{L}_2$-Stability Example

Magnetic Valve

Gap 7

Passivity-Based Control: Feedback Loop

Block Diagram

Gap 8

Conditions

- f is locally Lipschitz and $f(0, 0) = 0$
- h is continuous and $h(0, 0) = 0$

Design Goals

- Asymptotically stabilize passive system by feedback control
- Set asymptotically stable equilibrium point by feedback control

Passivity-Based Control: Asymptotic Stabilization

Theorem

Consider the feedback loop from above. Assume that the nonlinear system is passive with a radially unbounded positive definite storage function $V(x)$ and zero-state detectable. Let $\varphi(y)$ be a function such that $\varphi(0) = 0$ and $y^T\varphi(y)$ is positive definite. Then, the origin can be globally asymptotically stabilized by the feedback law $u = -\varphi(y)$.

Proof

Gap 9

Passivity-Based Control: Example

Interpretation

- Passive system can be made output strictly passive in order to achieve asymptotic stability
- Excess passivity is determined by choice of φ

Magnetic Valve

Gap 10

PCHD Systems: Definition

Port-Controlled Hamiltonian System with Dissipation (PCHD)

$$\dot{x} = f(x, u) = (J(x) - S(x))\frac{\partial V(x)}{\partial x}^T + G(x)u$$

- State $x \in \mathbb{R}^n$
- Input $u \in \mathbb{R}^m$
- Storage function $V(x)$ with $V(0) = 0$
- Input matrix $G(x)$
- Skew symmetric matrix $J(x) = -J^T(x)$
- Symmetric matrix $S(x) = S^T(x)$ positive semi-definite

$\Rightarrow$ Subclass of nonlinear systems that is suitable for modeling various mechanical systems

PCHD Systems: Example

Magnetic Valve with Input Current (instead of Voltage)

- $u = i_L^2$
- $z_1 = x_1, \; z_2 = m x_2$

State Equations

$$\dot{z}_1 = \frac{1}{m} z_2$$
$$\dot{z}_2 = \frac{1}{2}\frac{\partial L(z_1)}{\partial z_1} u - c z_1 - \frac{d}{m} z_2$$

Energy Function

$$V(x) = \frac{1}{2}(c z_1^2 + \frac{1}{m} z_2^2)$$

PCHD Representation

Gap 11

PCHD Systems: Passivity

Output Choice

$$y = G(x)^T \frac{\partial V(x)^T}{\partial x}$$

Computation

$$\begin{aligned}\dot{V}(x) &= \frac{\partial V(x)}{\partial x}\left((J(x) - S(x))\frac{\partial V(x)^T}{\partial x} + G(x)u\right)\\ &= \underbrace{\frac{\partial V(x)}{\partial x} J(x) \frac{\partial V(x)^T}{\partial x}}_{=0} - \underbrace{\frac{\partial V(x)}{\partial x} S(x) \frac{\partial V(x)^T}{\partial x}}_{\text{pos. semi-def.}} + \underbrace{\frac{\partial V(x)}{\partial x} G(x)}_{y^T} u\end{aligned}$$

Passivity of the System

$$y^T u = \dot{V}(x) + \frac{\partial V(x)}{\partial x} S(x) \frac{\partial V(x)^T}{\partial x} \geq \dot{V}(x)$$

PCHD Systems: Passivity-based Control

Goal

- Achieve asymptotic stability of a target equilibrium point $x_t \neq 0$

Synthesis Procedure

- Define targeted (autonomous) PHCD representation

$$\dot{x} = (J_t(x) - S_t(x)) \frac{\partial V_t^T(x)}{\partial x}$$

- Choose $V_t(x)$ such that $V_t(x_t) = 0$ and $V_t(x) > 0$ otherwise
- Choose damping injection matrix $S_d(x)$ positive semi-definite
- Use feedback

$$G(x)u = -G(x)S_d(x)G(x)^T \frac{\partial V_t^T}{\partial x} + (J_t(x) - S_t(x)) \frac{\partial V_t^T(x)}{\partial x} - (J(x) - S(x)) \frac{\partial V^T(x)}{\partial x}$$

$\Rightarrow$ x_t is asymptotically stable equilibrium point of the closed-loop system

$\Rightarrow$ $S_d(x)$ influences speed of convergence

PCHD Systems: Feedback Control Example

Magnetic Valve Example

Gap 12