Exercise Sheet 9:

Problem 20:

Consider the following nonlinear system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -ax_1^3 - kx_2 + u, \quad a, k > 0 \\ y &= x_2 \end{aligned}$$

a. Use the storage function $V(x) = \frac{1}{4}ax_1^4 + \frac{1}{2}x_2^2$ to show that the output y is output strictly passive

b. Show that the given system is finite-gain $\mathcal{L}_2$-stable and determine its maximum $\mathcal{L}_2$-gain.

c. Show that the given system is globally asymptotically stable for $u = 0$

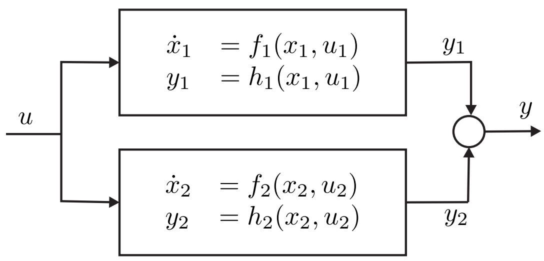
Problem 21:

We are given two nonlinear systems.

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, u_1) \\ y_1 &= h_1(x_1, u_1) \end{aligned} \qquad \begin{aligned} \dot{x}_2 &= f_2(x_2, u_2) \\ \dot{y}_2 &= h_2(x_2, u_2) \end{aligned}$$

It is assumed that the first system is output strictly passive with a storage function $V_1(x_1)$ and the second system is output strictly passive with a storage function $V_2(x_2)$.

Show that the parallel connection of both systems as shown in the following figure is also output strictly passive.



<u>Hint:</u> Recall that $y^T y = (y_1 + y_2)^T (y_1 + y_2) \leq 2(y_1^T y_1 + y_2^T y_2)$

Problem 22:

We are given the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -h(x_1) - ax_2 + u, \quad a > 0 \\ y &= \alpha x_1 + x_2, \quad 0 < \alpha < a \end{aligned}$$

a. Consider the storage function $V(x) = \frac{1}{2}ka^2x_1^2 + kax_1x_2 + \frac{1}{2}x_2^2 + \int_0^{x_1} h(\mu)d\mu$. Here, $k \in \mathbb{R}$ is a free parameter and the function $h : \mathbb{R} \to \mathbb{R}$ has the property that $h(\mu) \cdot \mu$ is positive definite. Choose an appropriate k to show that the system is output strictly passive.

b. Show that the system is globally asymptotically stable for $u = 0$