Exercise Sheet 8: La Salle's Theorem

Problem 17:

Consider the following nonlinear system:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + x_1^3 - x_2\end{aligned}$$

Determine the stability properties of the equilibrium point at the origin.

Problem 18:

Consider the following nonlinear system:

$$\begin{aligned}\dot{x}_1 &= \frac{2}{3}x_2 \\ \dot{x}_2 &= -x_1 + x_2(1 - 3x_1^2 - 2x_2^2)\end{aligned}$$

a. Show that the sets $S_1 = \{\begin{bmatrix}0\\0\end{bmatrix}\}$ and $S_2 = \{x | 1 - 3x_1^2 - 2x_2^2 = 0\}$ are invariant sets

Hint: For S_2, determine the time derivative of points on the circle and show that it fulfills the given nonlinear system equations. This means that trajectories starting on S_2 stay in S_2.

b. Study the stability of the set S_2

Hint: Choose an appropriate closed and bounded set M for the application of La Salle's theorem.

c. Study the stability of the set S_1

Hint: Use your result in **b.**

Problem 19:

We are given the following nonlinear system

$$\begin{aligned}\dot{x}_1 &= x_1^2 x_2 - 2x_2 - x_1 \\ \dot{x}_2 &= -x_1^3 - x_2\end{aligned}$$

a. Show that the invariant set for this nonlinear system consists of 3 equilibrium points

b. Use La Salle's theorem to show that the origin $x = 0$ is asymptotically stable

c. Use the result in **b.** to conclude that the remaining equilibrium points are instable

d. [optional] Verify your result by a phase plane analysis of the nonlinear system using Matlab (You can use the m-file `pplane7` that you can find on the web)