Exercise Sheet 7:

Problem 14:

The following nonlinear system is given

$$\dot{x}_1 = -x_1 - x_1 x_2^2$$
$$\dot{x}_2 = -x_2 - x_1^2 x_2$$

a. Show that the origin is the only equilibrium point of the nonlinear system

b. Use the variable gradient method to find a negative definite $\dot{V}(x)$ for the above nolinear system

Hint: Use the gradient function $\frac{\partial V(x)}{\partial x} = g(x) = \begin{bmatrix} ax_1 + bx_2 & bx_1 + cx_2 \end{bmatrix}$ and show that the parameters $a = 1$, $b = 0$ and $c = 1$ are suitable.

c. Compute $V(x)$ from $\frac{\partial V}{\partial x}$ in **b.** and show that $V(x)$ is positive definite

d. Assume that you found $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ and $\dot{V}(x) = -(x_1^2 + 2x_1^2 x_2^2 + x_2^2)$. Explain why the origin is globally asymptotically stable

Problem 15:

Consider the following nonlinear system:

$$\dot{x}_1 = x_1^3 + x_1^2 x_2$$
$$\dot{x}_2 = -x_2 + x_2^2 + x_1 x_2 + x_1^3$$

Show that the equilibrium point at the origin is instable.

Hint: Assume that the gradient of the function $V(x)$ is $\frac{\partial V(x)}{\partial x} = g(x) = \begin{bmatrix} ax_1 & bx_2 \end{bmatrix}$ and determine appropriate values for the parameters a and b.

Problem 16:

We are given the following nonlinear system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 - x_1 + x_1^3$$

a. Construct a Lyapunov function in order to show that the origin is an asymptotically stable equilibrium point

Hint: Try the gradient $g(x) = \begin{bmatrix} ax_1 + bx_2 + dx_1^3 & bx_1 + cx_2 \end{bmatrix}$ of $V(x)$

b. Estimate the region of attraction of the origin