Exercise Sheet 4: Describing Function

Problem 8:

Consider the van der Pol oscillator ($\mu > 0$)

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 + \mu x_2 - \mu x_1^2 x_2\end{aligned}$$

a. Rewrite the given state equations such that the describing function analysis can be applied. Sketch the corresponding block diagram representation

Hint: Try to express the nonlinearity $\psi(\cdot)$ in terms of the state variable x_1 and its time derivative $\dot{x}_1$

b. Verify that the nonlinearity $\psi(\cdot)$ fulfills all requirements for a describing function representation

c. Compute the describing function $N(a)$

Hint: Consider the trigonometric identities

$$\begin{aligned}\cos^3 x &= \frac{3\cos x + \cos 3x}{4}\\ \sin^2\theta &= 1 - \cos^2\theta\end{aligned}$$

and the integrals

$$\begin{aligned}\int \cos^2 x dx &= \frac{x}{2} + \frac{\sin 2x}{4}\\ \int \cos x \cos 3x dx &= \frac{\sin 2x}{4} + \frac{\sin 4x}{8}\end{aligned}$$

d. Predict the oscillation frequency and amplitude of the van der Pol oscillator

e. Predict the stability of the oscillation

Hint: Use the extended Nyquist criterion