Exercise Sheet 3: Phase Plane Analysis

Problem 5:

a. Consider the second-order autonomous system $\dot{x} = f(x)$ and its linearization $A(x_e)$ around an equilibrium point x_e. For each type of the equilibrium point of the linear system $\dot{x} = A(x_e)x$, determine if the equilibrium point x_e of the nonlinear system is stable or instable.

Equilibrium point of $\dot{x} = A(x_e)x$	Equilibrium point of $\dot{x} = f(x)$
stable node	
instable node	
stable focus	
instable focus	
center	
saddle	

b. Show that the equilibrium point $x_e = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ of the following nonlinear system is instable.

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2^2 \\ \dot{x}_2 &= x_2^3 + x_1^6 \end{aligned}$$

Problem 6:

Consider the following autonomous system

$$\begin{aligned} \dot{x}_1 &= x_1 - x_1^2 + x_2 \\ \dot{x}_2 &= -x_2 \end{aligned}$$

a. Sketch the phase plane plot of the above system in the range $-1.5 \leq x_1 \leq 1.5$ and $-1.5 \leq x_2 \leq 1.5$

b. Use the phase plant plot to decide about stability/instability of the equilibrium points of the system.

c. Use the Hartman-Grobman Theorem to verify your result in **b.**

Problem 7:

Assume that a nonlinear system $\dot{x} = f(x)$ with the initial condition x_0 is given and it is known that the solution fulfills $||x(t)|| < r$. What can you say about the behavior of the solution $x(t)$ depending on the dimension of the system and the existence of equilibrium points of the system?