Exercise Sheet 2

Problem 3:

Investigate the subsequent initial value problems regarding the following aspects:

- Type of the system (autonomous vs. non-autonomous)
- Existence of solutions
- Uniqueness of solutions
- Derivation of solutions in case of existence

$$\frac{d}{dt}x(t) = 1 + x(t)^2\,, \qquad x(0) = x_0 \tag{1}$$

$$\frac{d}{dt}x(t) = x(t)\,\sin x(t)\,, \qquad x(0) = x_0 \tag{2}$$

$$\frac{d}{dt}x(t) = -\mathrm{sgn}(x(t)) \qquad x(0) = 0 \tag{3}$$

$$\frac{d}{dt}x(t) = x(t)^{\frac{1}{3}}, \qquad x(0) = x_0 \tag{4}$$

$$\frac{d}{dt}x(t) = x(t)\,\sin t\,, \qquad x(0) = x_0 \tag{5}$$

Problem 4:

Determine the equilibrium points for the systems in equation (1) and (2). In addition, predict the basic system behavior around the respective equilibrium points.