Exercise Sheet 12: Flatness

Problem 28:

We are given the following nonlinear system

$$\begin{aligned}\dot{x}_1 &= -x_1^3 + \cos x_2 u\\ \dot{x}_2 &= \cos x_1 \cos x_2 + u\\ \dot{x}_3 &= x_2\end{aligned}$$

$$y = x_3$$

a. Determine the relative degree r for the system ouptut y. Where is the relative degree well-defined?

b. Determine a coordinates transformation $z = t(x)$ to Byrnes-Isidori canonical form

c. Verify that $z = t(x)$ fulfills all conditions for such coordinates transformation

d. Write down the system equations in the new z-coordinates

e. Compute the state feedback that linearizes the nonlinear system in the new z-coordinates and write down the system equations after applying this state feedback.

f. Determine the relative degree for the output $y = x_1$.

Problem 29:

We are given the third-order nonlinear system

$$\begin{aligned}\dot{x}_1 &= x_2 + 2x_1^2\\ \dot{x}_2 &= x_3 + u\\ \dot{x}_3 &= x_1 - x_3\\ y &= x_3\end{aligned}$$

a. Show that $y = x_3$ is a flat output of the nonlinear system

b. Determine a feedback control law that asymptotically stabilizes the point $y = 0$

Problem 30:

We consider the same system as in Problem 29 with the different output $y = x_1$.

a. Determine the relative degree of the system

b. Compute a state transformation to Byrnes-Isidori Canonical Form

c. Determine an asymptotically stabilizing state feedback for the input/ouput part of the nonlinear system

d. Is the overall system asymptotically stable?

Problem 31:

We consider the same system equations as in Problem 29 with the different output $y = x_1$ as in Problem 30.

a. Design a feedforward control law such that the output $y = x_1$ tracks the desired output

$$y_d(t) = \begin{cases} t^2 & \text{for } 0 \le t < 1\\ 2 - (t-2)^2 & \text{for } 1 \le t < 2\\ 2 & \text{for } t \ge 2\end{cases}$$

b. What do you expect for the behavior of the internal dynamics when the feedforward in **a.** is applied?

Hint: Investigate what happens for the internal dynamics if you use the feedforward control law computed in **a.**