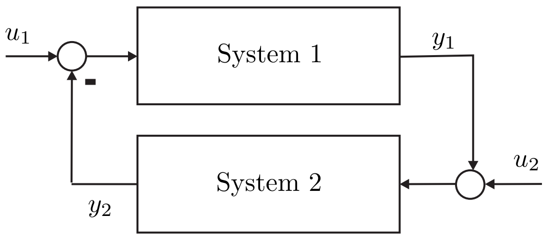
Exercise Sheet 10: Passivity-based Control

Problem 23:

We are given two nonlinear systems

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= -x_1^3 - x_2 + u_1 \\ y_1 &= x_2\end{aligned} \qquad\qquad \begin{aligned}\dot{z} &= -z + u_2 \\ y_2 &= z^3\end{aligned}$$



a. Show that the feedback connection of both nonlinear systems is passive

b. Is the feedback connection of both nonlinear systems finite-gain $\mathcal{L}_2$-stable?

c. Can you conclude that the origin $\begin{bmatrix} x_1 & x_2 & z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ is asymptotically stable for $u_1 = u_2 = 0$.

Problem 24:

We reconsider the nonlinear system from Problem 20

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -ax_1^3 - kx_2 + u, \quad a, k > 0 \\ y &= x_2\end{aligned}$$

Assume that $k = 20$.

a. Show that the system qualifies for the passivity-based feedback control design

b. Determine a feedback control law such that the excess of passivity is given by the function $100y^2$.

Problem 25:

We are given the following nonlinear system

$$\begin{aligned}\dot{x}_1 &= -8x_1x_2^3 \\ \dot{x}_2 &= x_1^2 - 8x_2^5 + 3x_1u\end{aligned}$$

a. Determine the PCHD representation of the nonlinear system

Hint: Try the storage function $V(x) = \frac{1}{2}x_1^2 + 2x_2^4$

b. Compute a feedback control law such that the closed-loop system has an asymptotically stable equilibrium point $x_e = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$